Viscous tilting and production of vorticity in homogeneous turbulence

M. Holzner,[1,*] M. Guala,[2] B. Lüthi,[1] A. Liberzon,[3] N. Nikitin,[4] W. Kinzelbach,[1] and A. Tsinober[3]
(International Collaboration for Turbulence Research)
[1] *Institute of Environmental Engineering, ETH Zurich, Wolfgang-Pauli-Str. 15, 8093 Zurich, Switzerland*
[2] *Galcit, California Institute of Technology, Pasadena CA 91125, USA*
[3] *School of Mechanical Engineering, Tel Aviv University, Ramat Aviv 69978, Israel*
[4] *Institute of Mechanics, Moscow State University, 119899 Moscow, Russia*
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Viscous depletion of vorticity is an essential and well known property of turbulent flows, balancing, in the mean, the net vorticity production associated with the vortex stretching mechanism. In this letter we however demonstrate that viscous effects are not restricted to a mere dissipation process, but play a more complex role in vorticity dynamics that is as important as vortex stretching. Based on results from particle tracking experiments (3D-PTV) and direct numerical simulation (DNS) of homogeneous and quasi isotropic turbulence, we show that the viscous term in the vorticity equation can also locally induce (i) production of vorticity, and (ii) changes of its orientation (viscous tilting). Such effects are not simply of kinematic nature. Indeed, while the dissipative nature of the viscous term may be recovered also in a random Gaussian velocity field, viscous production and tilting of vorticity represent genuine features of Navier Stokes turbulence, as emphasized by conditional statistics on the orientation between vorticity and the principal axis of the strain eigenframe.

In turbulent flows, the energy is injected at large scales by some forcing mechanism and dissipated into heat through the effect of viscosity at the smallest scales of motion, e.g. [1]. The main physical mechanisms that control fluid turbulence at the smallest scales are commonly described in terms of strain and vorticity, quantities that represent the tendency of fluid parcels to deform and rotate, respectively.

One of the most prominent processes occurring at small scales is the so-called 'vortex stretching': following a common argument [1], if a vortical fluid element is stretched by the surrounding flow, the rotation rate should increase to conserve angular momentum. However, Ref. [2] showed that this does not hold true point-wise and the dynamics are significantly influenced by a viscous contribution. The enstrophy balance equation,

$$\frac{D}{Dt}\frac{\omega^2}{2} = \omega_i\omega_j s_{ij} + \nu\omega_i\nabla^2\omega_i, \qquad (1)$$

where the squared vorticity magnitude ω^2 denotes the enstrophy, s_{ij} the rate of strain tensor and ν the kinematic viscosity of the fluid, contains a production term $\omega_i\omega_j s_{ij}$ and a viscous term $\nu\omega_i\nabla^2\omega_i$. The two terms in the mean approximately balance each other, i.e., $\langle\omega_i\omega_j s_{ij}\rangle \simeq -\langle\nu\omega_i\nabla^2\omega_i\rangle$ [1]. The presence of a viscous contribution in Eq. 1 shows that the effects of molecular viscosity are not limited to energy dissipation through deformation work, expressed as $\varepsilon = 2\nu s_{ij}s_{ij}$, but, among other things, it controls also vorticity growth. The effects of vortex stretching and viscous dissipation are usually captured in the well-known picture that in turbulence at small scales the nonlinearities increase gradients, whereas the viscosity depletes them, e.g. [1, 3] and references therein. However, as noted, e.g. in Ref. [1], viscous effects are not restricted to dissipation only. For example, viscosity may tilt vorticity, see, e.g. [1, 4–6] and is believed to be responsible for vortex reconnection, e.g. [4, 5]. It is reminded that this 'classical' reconnection mechanism (due to viscosity) is fundamentally different from reconnection events in quantum fluids, which take place due to a quantum stress acting at the scale of the vortex core without changes of total energy [7, 8]. However, direct evidence for the occurrence of tilting and production of vorticity due to viscosity is still missing in the literature, also because up to now it was difficult to measure the associated small scale quantities experimentally. Derivatives of the velocity became accessible through particle tracking experiments since the developments in, e.g. Ref. [2, 9, 10]. Ref. [10] recently measured viscous production of vorticity in proximity of turbulent/nonturbulent interfaces, which raised the question about the role of positive $\nu\omega_i\nabla^2\omega_i$ in fully developed and homogeneous turbulence.

In this letter we present the first measurements of tilting, depletion and considerable production of vorticity through viscosity in a turbulent flow through particle tracking velocimetry (Ref. [2, 9, 10]). The main goal is to unfold viscous effects on vorticity dynamics at the small scales of turbulence, with an emphasis on genuine (i.e. intrinsic to Navier Stokes turbulence as opposed to kinematic) effects. The results discussed hereafter are based on higher order derivatives and are challenging to obtain, both experimentally and numerically, which is why we compare the experimental results with those obtained through direct numerical simulation.

We measured the flow velocities and its gradients in a laboratory experiment of homogeneous, quasi isotropic and statistically stationary turbulence by using particle tracking velocimetry, see [2, 11] for details. Particle tracking velocimetry is based on high speed imaging of the motion of small buoyant tracer particles seeded into the flow. The experiment was carried out in a glass tank filled with water and the flow was forced mechanically from two sides by two sets of rotating disks as in Ref. [11]. The observation volume of approximately 15 x 15 x 20 mm^3 was centered with respect to the forced flow domain, mid-way between the disks. The turbulent flow is characterized by an r.m.s velocity of about 10 mm/s, a Taylor-based Reynolds number of $\mathrm{Re}_\lambda = 50$ and the Kolmogorov length and time scales are estimated at η =0.5 mm and τ_η =0.25 s, respectively. The Laplacian of vorticity, $\nabla^2\boldsymbol{\omega}$, is obtained indirectly from the local balance equation of vorticity in the form $\nabla \times \mathbf{a} = \nu\nabla^2\boldsymbol{\omega}$ by evaluating the term $\nabla \times \mathbf{a}$ from the Lagrangian tracking data. Through this indirect method only one derivative in space is needed instead of three, but particle positions have to be differentiated twice in time in order to get Lagrangian acceleration. For the numerical simulation we used an open source turbulence database[18] that was developed by Johns Hopkins University, see [12, 13] for details. The data are from a direct numerical simulation of forced isotropic turbulence on a 1024^3 periodic grid, using a pseudo-spectral parallel code. The Taylor Reynolds number is $Re_\lambda = 237$. After the simulation had reached a statistically stationary state, 1024 frames of data, which includes the 3 components of the velocity vector and pressure, were generated and stored into the database. The time interval covered by the data set is about one large-eddy turnover time. For comparison to a random velocity field, divergence-free Gaussian white noise was generated as in Ref. [14].

First, we statistically analyze effects of viscosity on the vorticity magnitude. One of the most basic phenomena of three dimensional turbulence is the predominant vortex stretching, which is manifested in a positive net enstrophy production, $\langle\omega_i\omega_j s_{ij}\rangle > 0$, e.g., [1, 3] and references therein. A strong positive skewness of the Probability Density Function (PDF) of the term $\omega_i\omega_j s_{ij}$ is indeed visible in Fig. 1, in agreement with earlier results, e.g. [3]. For statistically stationary turbulence the growth of enstrophy is balanced by viscous effects, i.e., the two terms on the RHS of Eq. 1 balance in the mean. Consistently, the term $\nu\omega_i\nabla^2\omega_i$ shows an opposite distribution, being strongly negatively skewed (Fig. 1). Although viscosity mostly depletes enstrophy, we note that also events where $\nu\omega_i\nabla^2\omega_i > 0$ are statistically significant. In fact, about one third of all events represent viscous production of enstrophy. The experimental curves qualitatively agree with the numerical ones, the $PDFs$ obtained from DNS are slightly more skewed. It is important to note that, while the reasons for the positiveness of the mean enstrophy production term are dynamical and due to interaction between vorticity and strain, the dissipative nature of the viscous term $\langle\nu\omega_i\nabla^2\omega_i\rangle < 0$ arises also for kinematical reasons: one can decompose the viscous term as, e.g.

$$\omega_\mathbf{i}\nabla^2\omega_\mathbf{i} = -\nabla\cdot(\omega\times(\nabla\times\omega)) - (\nabla\times\omega)^2, \qquad (2)$$

where the first term on the RHS is a divergence of a vector and vanishes in the mean for homogeneity, whereas the second is a (always negative) dissipation term [19]. Indeed, while for a Gaussian random field $\langle\omega_i\omega_j s_{ij}\rangle$=0 and the PDF of $\omega_i\omega_j s_{ij}$ becomes symmetric, the PDF of the viscous term is strongly negatively skewed, see the inset in Fig. 1. This means that the dissipative nature of the viscous term is also recovered in a random field and does not represent a genuine property of turbulent flow fields. However, from the same inset, we estimate that for a random gaussian field, the events with $\omega_i\nabla^2\omega_i > 0$ are statistically far less significant (about 2% of all events) compared to the same events in a Navier Stokes field (about 30%). We therefore conclude that considerable viscous production of vorticity is a genuine characteristic of Navier Stokes turbulence.

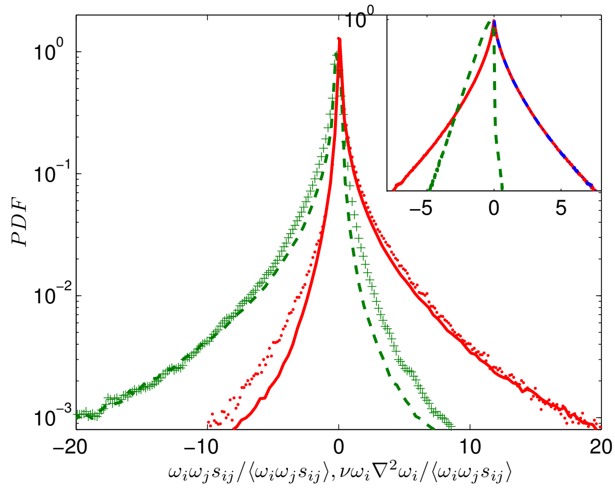

FIG. 1: $PDFs$ of $\omega_i\omega_j s_{ij}$ (—, •) and $\nu\omega_i\nabla^2\omega_i$ (− −, +) normalized with $\langle\omega_i\omega_j s_{ij}\rangle$. Symbols are from PTV, lines from DNS. The inset shows the analogous results from a random Gaussian velocity field, $\omega_i\omega_j s_{ij}$ (—), $\omega_i\nabla^2\omega_i$ (− −), the vertical reflection of the PDF corresponding to negative events, $\omega_i\omega_j s_{ij} < 0$ (− · −), demonstrates the symmetry.

The positiveness of the mean enstrophy production is associated with the predominant alignment between vorticity and the vortex stretching vector. The enstrophy production can be expressed as the scalar product of vorticity and the vortex stretching vector, $\omega_i\omega_j s_{ij} = \boldsymbol{\omega}\cdot\mathbf{W}$, where $W_i = \omega_j s_{ij}$. In real turbulent flows, the two vectors are strongly aligned. Thus, the PDF of the cosine between $\boldsymbol{\omega}$ and $\mathbf{W}$ is asymmetric (Fig.2a), in conformity with the prevalence of vortex stretching over vortex compression, whereas it is symmetric for a random Gaussian field (Fig. 2a), see also [3] and references therein. Anal-

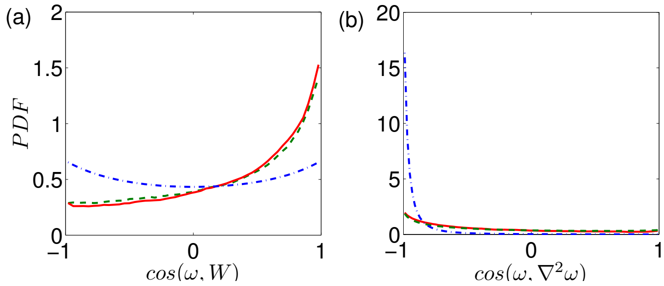

FIG. 2: $PDFs$ of the cosine between vorticity and the vortex stretching vector (a) and between vorticity and its Laplacian (b), as obtained from DNS (—), PTV (− −) and random Gaussian field (− · −).

ogously, we show the alignment between $\boldsymbol{\omega}$ and $\nabla^2\boldsymbol{\omega}$ in Fig. 2b. The figure shows high probabilities (much higher for the random field) of pronounced anti-alignment between $\boldsymbol{\omega}$ and $\nabla^2\boldsymbol{\omega}$, consistent with the negative skewness of the PDF of $\nu\omega_i\nabla^2\omega_i$, but we also note that with some smaller probability the two vectors can attain any orientation and, in particular, they can also be strongly aligned. This reminds of the results in [10], who measured $\cos(\boldsymbol{\omega},\nabla^2\boldsymbol{\omega}) \simeq 1$ in the proximity of the interface between turbulent and irrotational flow regions. The fact that the two vectors are not always strictly anti-aligned implies that the term $\nu\nabla^2\boldsymbol{\omega}$ does not act exclusively in the direction of the vorticity vector (mostly dampening and sometimes increasing the vorticity magnitude), but also normally to it, thus contributing to altering the orientation of vorticity. Since the negative skewness of the PDF is much stronger for the random velocity field than for the turbulent one, we may infer that viscous tilting is characteristic of fluid turbulence. The observation that the viscous term can effectively influence the orientation of vorticity is important, also because this will affect the relative orientation between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda_i}$ and therefore indirectly influence the vortex stretching (compression) mechanism.

The inviscid tilting of vorticity was measured by [6] and found to be sensitive to the alignments between vorticity and the strain eigenvectors. With the present data it is possible to estimate for the first time both the inviscid and the viscous contribution to the tilting of vorticity and to quantify the influence of the relative ($\boldsymbol{\omega}$-$\boldsymbol{\lambda_i}$) alignments. We adopt the approach of [6] and condition the data on situations of different alignment of vorticity with the principal axis of the strain eigenframe. Note that in a Gaussian field no differences are observed when conditioning on such alignments and therefore the expected effects in turbulent flow are explicitly dynamical.

Fig. 3a depicts the $PDFs$ of the two terms divided into the three subsets depending on the local alignment between $\boldsymbol{\omega}$ and $\boldsymbol{\lambda_i}$. The subsets are divided according to the condition $\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda_i}) \leq 0.7$, corresponding to a

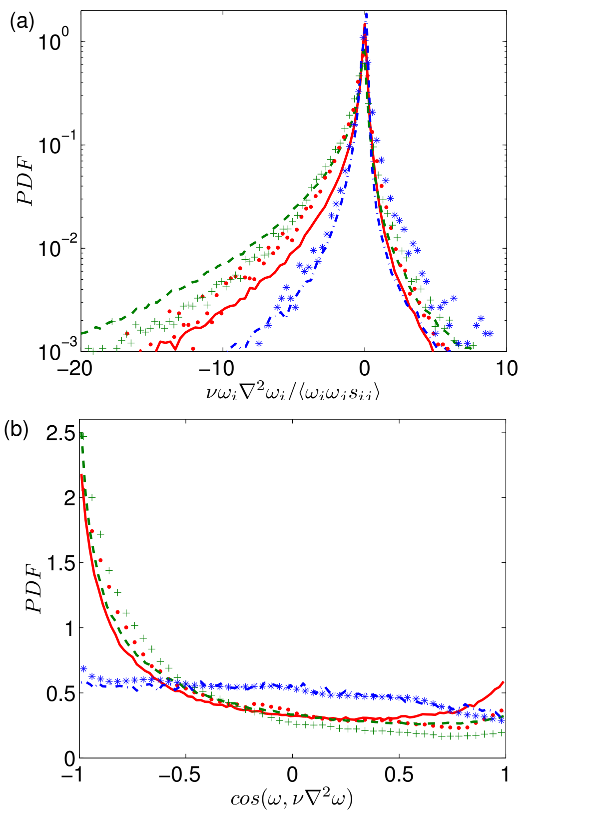

FIG. 3: $PDFs$ of $\omega_i\omega_j s_{ij}$ and $\nu\omega_i\nabla^2\omega_i$ (a) and of the cosine between vorticity and its Laplacian (b) for different ($\boldsymbol{\omega}$-$\boldsymbol{\lambda_i}$) alignments from DNS (lines) and PTV (symbols), $\boldsymbol{\omega}$ aligned with $\boldsymbol{\lambda_1}$ (—, •), $\boldsymbol{\lambda_2}$ (− − , +) and $\boldsymbol{\lambda_3}$ (− · −, ∗).

cone of roughly 33°, as in [6]. It is visible that, while for the case of alignment with the intermediate eigenvector, $\boldsymbol{\lambda_2}$, the PDF becomes more skewed, i.e. $\nu\omega_i\nabla^2\omega_i$ contributes more to the reduction of ω^2, whereas in the case of alignment with $\boldsymbol{\lambda_1}$, the skewness decreases and even more so when vorticity is aligned with $\boldsymbol{\lambda_3}$. Again, the main qualitative trends are the same both for the numerical and experimental results, with the curves obtained from DNS showing a stronger skewness. In Fig. 3b we analyze how this qualitatively different behavior of the term $\nu\omega_i\nabla^2\omega_i$ is reflected in the alignment between $\boldsymbol{\omega}$ and $\nabla^2\boldsymbol{\omega}$. The PDF of the cosine between the two vectors is strongly negatively skewed for the cases when $\boldsymbol{\omega}$ is aligned with $\boldsymbol{\lambda_1}$ and $\boldsymbol{\lambda_2}$. In the case of $\boldsymbol{\omega}$ aligned with $\boldsymbol{\lambda_3}$ the distribution changes dramatically becoming very flat in conformity with the reduced skewness of the PDF of $\nu\omega_i\nabla^2\omega_i$. Therefore, in this case viscosity contributes less to the dissipation of enstrophy, but still plays a role, e.g. for the tilting of the vorticity vector.

The inviscid and the viscous contribution to the total tilting $\boldsymbol{\Omega}$ of vorticity can be written as follows,

$$\Omega_k = \frac{D\widehat{\omega}_k}{Dt} = \eta^i_{\omega_k} + \eta^v_{\omega_k}, \tag{3}$$

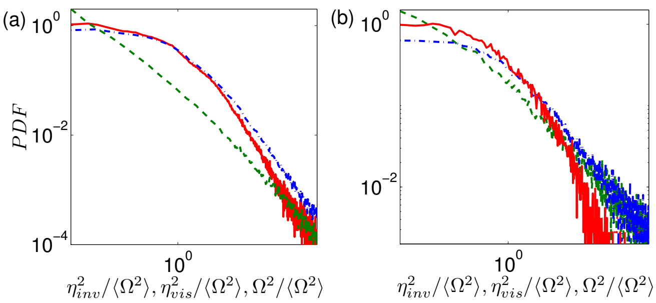

FIG. 4: PDFs of inviscid (—), viscous (− −) and total (− · −) tilting from DNS (left) and PTV (right).

where $\eta^i_{\omega_k} = \frac{\omega_j s_{kj}}{\omega} - \frac{\omega_l \omega_j s_{lj}}{\omega^3}\omega_k$ and $\eta^v_{\omega_k} = \frac{\nu\nabla^2\omega_k}{\omega} - \frac{\nu\omega_j\nabla^2\omega_j}{\omega^3}\omega_k$ represent the inviscid and viscous tilting respectively. Fig. 4 shows PDFs of the squared magnitudes of total, inviscid and viscous tilting and it appears that viscous tilting is typically smaller than the inviscid one, but at large magnitudes both contributions to the total tilting are comparably significant. The $PDFs$ of viscous and total tilting obtained from PTV appear to be somewhat higher at the tails compared to the numerical result, but the experimental scatter is considerable at high magnitudes. In order to appreciate the dependence of the tilting magnitudes on geometrical properties introduced before, it is useful to write the following equations,

$$(\eta^i_\omega)^2 = \sqrt{\mathbf{W}^2/\omega^2} sin^2(\boldsymbol{\omega}, \mathbf{W}) \quad (4)$$
$$= \Lambda_k^2 cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda_k}) - (\Lambda_k cos^2(\boldsymbol{\omega}, \boldsymbol{\lambda_k}))^2 \quad (5)$$

and

$$(\eta^v_\omega)^2 = \sqrt{(\nu\nabla^2\boldsymbol{\omega})^2/\omega^2} sin^2(\boldsymbol{\omega}, \nabla^2\boldsymbol{\omega}). \quad (6)$$

From Eq. 5 one can see that the inviscid tilting vanishes identically, when $\boldsymbol{\omega}$ is strictly aligned with $\boldsymbol{\lambda_i}$. This alignment can then only be changed in two ways: through viscous tilting and/or through a change of the orientation of strain eigenframe.

In summary, in this letter we have shown that viscosity in two thirds of all events depletes enstrophy and that this effect is essentially of kinematic nature. Viscous tilting and production of vorticity, which occur in one third of all events, are instead characteristic features of turbulent flows. Our results demonstrate that viscosity influences enstrophy production by changing vorticity in magnitude and direction. The observed effects are sensitive to the ($\boldsymbol{\omega}$-$\boldsymbol{\lambda_i}$) alignments and thus to the local vortex stretching (compression) regime. When $\boldsymbol{\omega}$ is aligned with $\boldsymbol{\lambda_3}$ the purely dissipative contribution of $\nu\omega_i\nabla^2\omega_i$ is strongly suppressed. From the technical point we note that the experimental and numerical results agree well with each other on the qualitative level. Some quantitative discrepancies might be attributed to the fact that experimental measurements are affected by limited spatial resolution, noise and to the difference in Reynolds numbers. Finally, we propose a plausible postulate regarding the role of $\nu\omega_i\nabla^2\omega_i$ for the predominant $\boldsymbol{\omega}$-$\boldsymbol{\lambda_2}$ alignment so typical for turbulent flows, e.g. [15, 16]. In these situations the vectors $\boldsymbol{\omega}$ and $\nabla^2\boldsymbol{\omega}$ are predominantly anti-aligned. Strong $\boldsymbol{\omega}$-$\boldsymbol{\lambda_2}$ alignment stalls inviscid tilting, while the anti-alignment of $\boldsymbol{\omega}$ and $\nabla^2\boldsymbol{\omega}$ points to reduced viscous tilting, i.e., both mechanisms could work towards maintaining the alignment. In future work we hope to pursue these questions that are intimately related to moderation of enstrophy growth and to prevention of finite time singularities [17]. This will also require to address the tilting mechanisms of the strain eigenframe.

* holzner@ifu.baug.ethz.ch

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[19] It is noteworthy that the above decomposition of $\nu\omega_i\nabla^2\omega_i$ - though useful - has a limitation since it is not unique and there is an infinite number of possibilities to represent it as a sum of a dissipation and a flux term (i.e. as a divergence of some vector). There is no way to define dissipation (i.e. to choose one among many purely negative expressions) of enstrophy as it is not an inviscidly conserved quantity, unlike the kinetic energy [3].